SOUTHERN CALIFORNIA FUNCTIONAL ANALYSIS SEMINAR

Saturday, April 18, 2009
2:00 – 4:30 PM

Claremont McKenna College
Davidson Lecture Hall, Adams Hall, Lower Level
(See reverse for map and directions to campus)

2:00 – 3:00 PM, Lecture #1:

UNIFORM CONTINUITY AND UNIFORM CONVERGENCE REVISITED

GERALD BEER
CALIFORNIA STATE UNIVERSITY, LOS ANGELES

Let B be an ideal of subsets of a metric space $\langle X, d \rangle$. This paper considers a strengthening of the notion of uniform continuity of a function restricted to members of B which reduces to ordinary continuity when B consists of the finite subsets of X and agrees with uniform continuity on members of B when B is either the power set of X or the family of compact subsets of X. The paper also presents new function space topologies that are well-suited to this strengthening. As a consequence of the general theory, we display necessary and sufficient conditions for continuity of the pointwise limit of a net of continuous functions.

References

1. R. Arens, *A topology for spaces of transformations*, Ann. of Math. **47** (1946), 480-495.
2. M. Atsuji, *Uniform continuity of continuous functions of metric spaces*, Pacific J. Math. **8** (1958), 11-16.
3. G. Beer and S. Levi, Strong uniform continuity, J. Math. Anal. Appl. **350** (2009), 568-589.
4. H. Hogbe-Nlend, *Bornologies and functional analysis*, North-Holland, Amsterdam, 1977.

3:30 – 4:30 PM, Lecture #2:

HOW MANY SUBSPACES CAN AN OPERATOR SPACE HAVE?

TIMUR OIKHBERG
UNIVERSITY OF CALIFORNIA, IRVINE

The first half of the talk will be expository: I will present an overview of the known results on the complexity of various "naturally occurring" relations (such as isometry or isomorphism) between infinite dimensional subspaces of a given separable Banach space. Here, the term "complexity" is understood to mean Borel reducibility to certain "classical" equivalence relations on Polish spaces. I then introduce operator spaces, and present an operator space with "relatively few" subspaces. More precisely, the relation of complete isomorphism on the subspaces of this operator space is Borel bireducible to the complete K_σ relation.

This is a joint work with C. Rosendal.

Dinner at a local restaurant will follow the concluding lecture.
For more information, please contact Professor Asuman Aksoy at (909) 607-2769, or via email at asuman.aksoy@cmc.edu.

SOUTHERN CALIFORNIA FUNCTIONAL ANALYSIS SEMINAR

Saturday, November 15, 2008
2:00 – 4:30 PM

Claremont McKenna College
Davidson Lecture Hall, Adams Hall, Lower Level
(See reverse for map and directions to campus)

2:00 – 3:00 PM, Lecture #1:

DISTINGUISHED VARIETIES: DETERMINANTAL REPRESENTATIONS AND BOUNDED EXTENSIONS

GREG KNESE
UNIVERSITY OF CALIFORNIA, IRVINE

Distinguished varieties are algebraic curves in C^2 that exit the unit bidisk through the distinguished boundary. We will discuss how these curves appear naturally in operator theory and function theory, and we will outline a connection between distinguished varieties and polynomials with no zeros on the bidisk (on the surface, two antithetical objects) that allows us to prove a determinantal representation and a "bounded analytic extension theorem" for distinguished varieties.

3:30 – 4:30 PM, Lecture #2:

HYPERGROUPS AND PROBABILITY THEORY

HERBERT HEYER
TUEBINGEN, GERMANY

Hypergroups are locally compact spaces with a group-like structure for which the bounded measures convolve in a similar way to that of a locally compact group. Important examples of hypergroups are orbit spaces arising from groups.

There are fundamental constructions providing hypergroup structures on the nonnegative reals (Sturm - Liouville functions) and on the nonnegative integers (Jacobi polynomials).

In probability theory hypergroup convolutions admit for example the study of invariant Markov chains and Levy processes, prominent results being (local) central limit theorems and martingale characterizations respectively.

The method of carrying out the analysis of hypergroups and their applications is a generalization of the Fourier transform of measures defined on a dual object attached to the given hypergroup.

Dinner at a local restaurant will follow the concluding lecture.
For more information, please contact Professor Asuman Aksoy at (909) 607-2769, or via email at asuman.aksoy@cmc.edu.

SOUTHERN CALIFORNIA
FUNCTIONAL ANALYSIS SEMINAR

Saturday, April 28, 2007
2:00 – 4:30 PM

Claremont McKenna College
Davidson Lecture Hall, Adams Hall, Lower Level

2:00 – 3:00 PM, Lecture #1:

TWO-NORM SPACES AND SOME OF THEIR APPLICATIONS

P. K. SUBRAMANIAN

CALIFORNIA STATE UNIVERSITY, LOS ANGELES

The theory of two-norm spaces was invented by the Polish School of Functional Analysts in the fifties. In this talk we consider some of the applications of the theory.

3:30 – 4:30 PM, Lecture #2:

ISOMETRIES OF NONCOMMUTATIVE BANACH SPACES

DAVID SHERMAN

UNIVERSITY OF CALIFORNIA, SANTA BARBARA

Many Banach spaces are described most naturally as spaces of functions. Motivated partially by quantum mechanics, their noncommutative versions are built out of *operators*. Examples are C^*-algebras, von Neumann algebras, and noncommutative L^p spaces.

In this talk I will start with classical results of Banach on isometries of $C(K)$ and L^p spaces and I will explain their evolution into fully noncommutative theorems. Although new phenomena have appeared and the proofs have gotten more complicated, there is a surprising unity (and simplicity) in the results: all of these isometries should be viewed as "noncommutative weighted composition operators."

Dinner at a local restaurant will follow the concluding lecture.
For more information, please contact Professor Asuman Aksoy at (909) 607-2769, or via email at asuman.aksoy@cmc.edu.

SOUTHERN CALIFORNIA FUNCTIONAL ANALYSIS SEMINAR

Saturday, November 17, 2007
2:00 – 4:30 PM

Claremont McKenna College
Davidson Lecture Hall, Adams Hall, Lower Level

2:00 – 3:00 PM, Lecture #1:

GLOBAL STABILITY FOR CONTINUOUS AND DISCRETE DYNAMICAL SYSTEMS

MARIO MARTELLI
CLAREMONT McKENNA COLLEGE

Two conjectures on global asymptotic stability have been recently disproved. The first conjecture, due to Markus-Yamabe (1960), regards continuous and autonomous systems of Differential Equations. The second, due to La Salle (1976), regards autonomous systems of Difference Equations. A counterexample to the first was found in 1997 and a counterexample to the second was discovered in 1998.

However, the story is not that simple. Global asymptotic stability of an equilibrium point can be obtained in the case discussed by Markus-Yamabe when the function that governs the system is continuous, its Gateaux derivative exists except possibly on a linearly countable set S, and the spectrum of the symmetric part of the derivative is strictly contained in the left hand side of the real line.

Similarly, global asymptotic stability can be proved in the case discussed by La Salle when the function is continuous, Gateaux differentiable except possibly on S, and the spectral radius of the matrix obtained by multiplying the Gateaux derivative $F'_G(\mathbf{x})$ with its transpose is strictly smaller than 1.

In this talk I shall present the counterexamples and the proof of the two positive outcomes. I shall also show that the set S cannot be uncountable, even in the case when its Lebesgue measure is 0.

3:30 – 4:30 PM, Lecture #2:

MULTI-BANACH SPACES AND MULTI-BANACH ALGEBRAS

H. G. DALES
UNIVERSITY OF LEEDS, UNITED KINGDOM

I have developed a theory of "multi-Banach spaces"; this involves a sequence of norms on the spaces E^n, where E is a Banach space. The theory is somewhat related to that of operator spaces - but technically has no overlap.

First it gives a new way of looking at the geometry of Banach spaces. Second a key example involves Banach lattices, and so we can generalize some results from that subject. Third we discuss "multi-continuous" linear operators, and define some new (classical) Banach algebras of operators. Fourth, we can give a new abstract notion of orthogonality. Finally, we can formulate an obvious notion of a "multi-Banach algebra", bringing in a generalization of the group algebra $L^1(G)$, and resolve at least one classical problem connected with amenability.

Dinner at a local restaurant will follow the concluding lecture.
For more information, please contact Professor Asuman Aksoy at (909) 607-2769, or via email at asuman.aksoy@cmc.edu.

SOUTHERN CALIFORNIA FUNCTIONAL ANALYSIS SEMINAR

on

Saturday, March 25, 2006

2:00 – 4:30 PM

Claremont McKenna College

Davidson Lecture Hall, Adams Hall, Lower Level

<u>2:00 – 3:00 PM, Lecture #1:</u>

A NATURAL SHARP CHARACTERIZATION OF A^1

WINSTON OU
Scripps College

By considering a natural generalization of the Hardy-Littlewood maximal operator, we obtain a new, sharp characterization of A^1 weights in terms of their A^∞ numbers and the BLO norms of their logarithms."

<u>3:30 – 4:30 PM, Lecture #2:</u>

CONVOLUTION ALGEBRAS ON R^+

SANDY GRABINER
Pomona College

The classical convolution algebras on $R^+ = [0,\infty)$ are the algebra of integrable functions and the analogous algebra of measures. For the past 30 years there has also been substantial study of weighted versions of these algebras. These weighted algebras are fundamental building blocks in the theory of Banach algebras, particularly radical Banach algebras, and also are the natural domain for the operational calculus for continuous semigroups of operators. We will describe results about ideals, convergence, semigroups, and homomorphisms of these algebras. Many of these results are new even in the classical case. In particular, we will discuss recent results that make substantial use of the weak* topology.

Dinner at a local restaurant will follow the concluding lecture.

For more information, please contact Professor Asuman Aksoy at (909) 607-2769, or via email at asuman.aksoy@claremontmckenna.edu.

SOUTHERN CALIFORNIA FUNCTIONAL ANALYSIS SEMINAR

Saturday, October 28, 2006
2:00 – 4:30 PM

Claremont McKenna College
Davidson Lecture Hall, Adams Hall, Lower Level

2:00 – 3:00 PM, Lecture #1:

COMPLEX SYMMETRIC OPERATORS

STEPHAN GARCIA (POMONA COLLEGE)

Roughly stated, a linear operator on a complex Hilbert space is complex symmetric if it has a symmetric matrix representation (with complex entries) with respect to some orthonormal basis. This surprisingly large class of operators includes all normal operators, Hankel operators, compressed Toeplitz operators (e.g. finite Toeplitz matrices), and many integral and differential operators (e.g. the Volterra integration operator). We will discuss several examples and highlight a few recent structure theorems for this class.

3:30 – 4:30 PM, Lecture #2:

BEURLING-MALLIAVIN THEORY FOR SUB-EXPONENTIAL AND SUPER-EXPONENTIAL GROWTH

NIKOLAI MAKAROV
CALTECH

Let $\rho > 0$ and let $\gamma : \mathbb{R} \to \mathbb{R}$ be a smooth function satisfying

$$\gamma'(x) \gtrsim -|x|^{\rho-1}, \quad (x \to \pm\infty).$$

Consider the Toeplitz operator with symbol $u = e^{i\gamma}$,

$$T[u] : H^2(\mathbb{R}) \to H^2(\mathbb{R}), \quad f \mapsto P_+(uf).$$

Here $H^2(\mathbb{R})$ is the Hardy space and $P_+ : L^2(\mathbb{R}) \to H^2(\mathbb{R})$ is the orthogonal projection. We establish a criterion for the injectivity of $T[u]$, up to multiplication of the symbol by an arbitrarily small power of the "gap" factor

$$\{x \in \mathbb{R} : \forall t \geq x,\ \gamma(\mathrm{t}) \leq \gamma(x)\}.$$

In the statements below, the sums are taken over intervals in $\mathcal{BM}(\gamma)$, l denotes the length of the interval, and d its distance from the origin plus *l*.

Theorem (sub-exponential case $\rho \leq 1$**).**

(i) *If* $\gamma \notin (*)$, *or if* $\gamma \in (*)$ *but* $\sum d^{-2} l^2 = \infty$, *then* $\ker T\left[uM^{\in}\right] = 0$ *for all* $\in > 0$.

(ii) *If* $\gamma \in (*)$ *and* $\sum d^{-2} l^2 < \infty$, *then* $\ker T\left[uM^{\in}\right] \neq 0$ *for all* $\in < 0$.

Theorem (super-exponential case $\rho \geq 1$**).**

(i) *If* $\gamma \notin (*)$, *or if* $\gamma \in (*)$ *but* $\sum d^{\rho-3} l^2 = \infty$, *then* $\ker T\left[uM^{\in}\right] = 0$ *for all* $\in > 0$.

(ii) *If* $\gamma \in (*)$ *and* $\sum d^{\rho-3} l^2 < \infty$, *then* $\ker T\left[uM^{\in}\right] \neq 0$ *for all* $\in < 0$.

In the exponential case $\rho = 1$, the choice $U = B\overline{S}$, where $S(x) = e^{ix}$ and B is a Blaschke product, gives the classical BM theorem about completeness of exponentials in $L^2(a,b)$. Our results have the same realm of applications as the BM theory (completeness and minimality of families of special functions, weighted approximation, distribution of zeros of entire functions, gap and density theorems, direct and inverse spectral problems), but these applications concern the theory of Fourier transforms associated with general self-adjoint problems (perhaps with singular endpoints) rather than just the theory of classical Fourier transform.

This is a joint work with Alexei Poltoratski (Texas A&M).

Dinner at a local restaurant will follow the concluding lecture.
For more information, please contact Professor Asuman Aksoy at (909) 607-2769, or via email at asuman.aksoy@cmc.edu.

SOUTHERN CALIFORNIA FUNCTIONAL ANALYSIS SEMINAR

on

Saturday, September 17, 2005
1:30 – 4:00 PM

Claremont McKenna College
Davidson Lecture Hall, Adams Hall, Lower Level

<u>1:30 – 2:30 PM, Lecture #1:</u>

“RANDOM” PRODUCTS OF PROJECTIONS IN HILBERT SPACE

RONALD E. BRUCK

One of the algorithms used in computed tomography is the “product of projections” theorem, which dates back to von Neumann: if P_1, P_2, . . . , P_n are orthogonal projections from a Hilbert space H onto (necessarily closed) subspaces $R(P_i)$, then

$$(P_1P_2 : : : P_N)_n x \ ! \ y \ (n \ ! \ 1);$$

where $y \ 2 \ \backslash_i R(P_i)$. (Von Neumann proved this for $N = 2$.)

Considerably more is known: in 1965, Amemiya and Ando proved the astonishing result that the projections don’t have to be iterated in *cyclic* order: they can be iterated in any, arbitrary, “random” order, and so long as each is reused infinitely often, the resulting sequence will converge *weakly* to a common fixed-point. They left open the question of whether such an iteration must converge *strongly*.

And there, exactly forty years later, the problem remains. But going back to ideas of Halperin (1962), who proved the general cyclic convergence, and new methods from the theory of semi-definite programming, we hope to show that the solution of this problem is tantalizingly close. And we conjecture that the answer is *Yes, the iteration converges strongly*. But it does seem that elementary linear algebra will provide the proof, not any fancy functional analysis.

<u>3:00 – 4:00 PM, Lecture #2:</u>

FROM STATIONARY TO HARMONIZABILITY

M.M. RAO

If f is a real integrable function on a measure space $(X; \ ^1)$ and T is a measurable and measure preserving transformation, let $f; \ Tf; \ T_2f; \ :::; \ f_k = T_kf$ be iterates of f under T. Then in 1931 G.D. Birkhoff proved that the sequence of arithmetical means

$$(f + f_1 + ::: + f_n)=n$$

converges pointwise a.e. to $f_{¤}$ and the limit is invariant, $Tf_{¤} = f_{¤}$.
The limit is a constant if T is ‘ergodic’, and ^{1} is finite. The proof was difficult to follow, and the following year A. I. Khintchine has given a detailed argument, and also introduced two generalizations of the f_n-sequence above, called the strict and weak sense stationarities, which later played crucial roles in many applications. In 1947 M. Loeve generalized one of the classes, and a final extension was introduced by S. Bochner in 1955. These are called harmonizable families.
In this talk, all these classes will be discussed in some detail, and their Fourier integral representations as well as a few related new developments will be indicated.

Dinner at a local restaurant will follow the concluding lecture.
For more information, please contact Professor Asuman Aksoy at (909) 607-2769, or via email at Asuman.Aksoy@claremontmckenna.edu.

SOUTHERN CALIFORNIA FUNCTIONAL ANALYSIS SEMINAR

on

Saturday, November 12, 2005
2:00 – 4:30 PM

Claremont McKenna College
Davidson Lecture Hall, Adams Hall, Lower Level

(See reverse for map and directions to campus)

<u>2:00 – 3:00 PM, Lecture #1:</u>

MATHEMATICAL CONCEPTUALISM

NIK WEAVER
WASHINGTON UNIVERSITY, SAINT LOUIS

Is circular reasoning necessary in mathematics? Many mainstream mathematicians might be surprised to learn that the standard axiomatizations of set theory involve fundamental circularities. Mathematical conceptualism is an alternative foundational philosophy, originating in views of Poincare and Russell, which strictly forbids all circularity. This sort of approach was originally thought to be far too weak to support ordinary core mathematics, and later was felt to be subject to severe limitations of a more abstract nature, but we now know that these limitations are not valid and in fact essentially all core mathematics is conceptualistically legitimate if interpreted properly. At the same time, conceptualism exorcises vast regions of set-theoretic pathology from the mathematical universe, so that it is in fact in better accord with actual mathematical practice than the Cantorian picture. I believe a strong case can be made for abandoning Cantorian set theory as a foundation for mathematics, and adopting conceptualism in its place.
(Related papers are available at http://www.math.wustl.edu/~nweaver/conceptualism.html)

<u>3:30 – 4:30 PM, Lecture #2:</u>

ANALYTIC CAPACITY, LIPSCHITZ HARMONIC CAPACITY, BILIPSCHITZ MAPS AND CANTOR SETS

JOHN GARNETT
UNIVERSITY OF CALIFORNIA, LOS ANGELES

Let $E \subset \mathbb{C}$ be a compact plane set. The **analytic capacity** of E is

$$\gamma(E) = \sup\left\{ \left| f'(\infty) \right| : f \text{ is analytic on } \mathbb{C} \setminus E \text{ and } \sup_{\mathbb{C}\setminus E} \left| f(z) \right| \le 1 \right\} \quad \text{where} \quad f'(\infty) = \lim_{z\to\infty} z\left(f(z) - f(\infty) \right).$$

Thus $\gamma(E) > 0$ if and only if $\mathbb{C} \setminus E$ supports a non-constant bounded analytic function. The general problem is to find a geometric characterization of sets of positive analytic capacity. Classical theorems of Riemann say that $\gamma(E) = 0$ if E is finite and $\gamma(E) > 0$ if E is infinite and connected. It is also known that $\gamma(E) = 0$ if E has zero one-dimensional measure and that $\gamma(E) > 0$ if the Hausdorff dimension $\dim(E) > 1$. Recent exciting work of Tolsa, using ideas of Melnikov, Verdera, David and many others, shows that $\gamma(E) > 0$ if and only if E supports a positive measure μ with $\mu(B(z, r)) \le r$ for every ball $B(z, r)$ that has finite **Menger curvature**

$$\iiint \frac{1}{R(z,w,\zeta)^2}\, d\mu(w)\, d\mu(\zeta)\, d\mu(z) < \infty \quad \text{where } R(z,w,\zeta) \text{ denotes the radius of the circle through } z, w \text{ and } \zeta.$$

A homeomorphism $T : E \to T(E)$ is **bilipschitz** if T and T^{-1} satisfy Lipschitz conditions

$$\frac{1}{K}\left| z - w \right| \le \left| T(z) - T(w) \right| \le K\left| z - w \right|$$

for all $z, w \in E$, Tolsa has most recently proved that if T is bilipschitz, then $\gamma(T(E)) \le C(K)\gamma(E)$, where $C(K)$ depends only on the constant K. The *Lipschitz harmonic capacity* of E is the $\mathbb{R}^d$ analogue of the analytic capacity of E, and much present work concerns extending results about analytic capacity to the case of Lipschitz harmonic capacity. Here, a big problem is to avoid Menger curvature. The talk will give an introduction to Lipschitz harmonic capacity, describe work of Volberg and others and unsolved problems about Lipschitz harmonic capacity.

Dinner at a local restaurant will follow the concluding lecture.
For more information, please contact Professor Asuman Aksoy at (909) 607-2769, or via email at Asuman.Aksoy@claremontmckenna.edu.

MAP & DIRECTIONS TO CAMPUS

I-10 WESTBOUND (from San Bernardino)

Stay on I-10 West (toward Los Angeles) until you reach the **Indian Hill/Claremont** exit. Turn right (north) off the exit. You will be on Indian Hill; continue north on Indian Hill for about 1.5 miles until you reach **10th Street.** Turn right on 10th Street and follow it until it ends on **Columbia.** Turn right (south) on Columbia, then left (east) on **9th Street**. Park anywhere on 9th Street. Adams Hall is on the south side of the street, and Davidson Lecture Hall is on the southwest side of the building, on the lower level.

I-10 EASTBOUND (from Los Angeles)

Stay on the I-10 East (toward San Bernardino) until you reach the **Indian Hill/Claremont** exit. Turn left (north) off the exit. You will be on Indian Hill; continue north on Indian Hill for about 1.5 miles until you reach **10th Street.** Turn right on 10th Street and follow it until it ends on **Columbia.** Turn right (south) on Columbia, then left (east) on **9th Street**. Park anywhere on 9th Street. Adams Hall is on the south side of the street, and Davidson Lecture Hall is on the southwest side of the building, on the lower level.

I-210 WESTBOUND (from San Bernardino)

Stay on I-210 West (towards Pasadena) until you reach the **Towne Avenue** exit. Turn left off the exit. You will be on Towne; continue south for about one mile until you reach **Foothill Boulevard**. Turn left on **Foothill Boulevard**. Continue east on Foothill for about one mile and turn right onto **Dartmouth Avenue**. Continue south on Dartmouth for three blocks to **10th Street** and turn left. Follow 10th Street to **Columbia** and turn right. Then turn left (east) on **9th Street** and park anywhere on 9th Street. Adams Hall is on the south side of the street, and Davidson Lecture Hall is on the southwest side of the building, on the lower level.

I-210 EASTBOUND (from Pasadena)

Stay on the I-210 East (towards San Bernardino) until you reach the **Towne Avenue** exit. Turn right off the exit. You will be on Towne; continue south for about one mile until you reach **Foothill Boulevard**. Turn left on **Foothill Boulevard**. Continue east on Foothill for about one mile and turn right onto **Dartmouth Avenue**. Continue south on Dartmouth for three blocks to **10th Street** and turn left. Follow 10th Street to **Columbia** and turn right. Then turn left (east) on **9th Street** and park anywhere on 9th Street. Adams Hall is on the south side of the street, and Davidson Lecture Hall is on the southwest side of the building, on the lower level.

 # 5 on map below: